21. $$\int_{-\infty}^{\infty} e^{-|x|}dx = \int_{-\infty}^{0} e^{-|x|}dx + \int_{0}^{\infty} e^{-|x|}dx$$

$$= \lim_{b\to-\infty}\int_{b}^{0} e^{-|x|}dx + \lim_{b\to\infty}\int_{0}^{b} e^{-|x|}dx$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\lim_{b\to-\infty}\int_{b}^{0} e^{-(-x)}dx + \lim_{b\to\infty}\int_{0}^{b} e^{-x}dx$$

$$\lim_{b\to-\infty} e^{x}\Big|_{b}^{0} + \lim_{b\to\infty} -e^{-x}\Big|_{0}^{b}$$

$$\lim_{b\to-\infty}\left(e^{0}-e^{b}\right) + \lim_{b\to\infty}\left(-e^{-b}- -e^{-0}\right)$$

$$1-0 \quad + \quad 0+1$$

$$2$$

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23 $$\int \frac{1}{e^{x}+e^{-x}}dx = \int \frac{1\,e^{x}}{\left(e^{x}+\frac{1}{e^{x}}\right)e^{x}}dx$$

$$= \int \frac{e^{x}\,dx}{(e^{x})^{2}+1} \qquad u = e^{x}, \quad du = e^{x}dx$$

$$= \int \frac{du}{u^{2}+1} = \tan^{-1}u$$

$$= \tan^{-1}e^{x}$$

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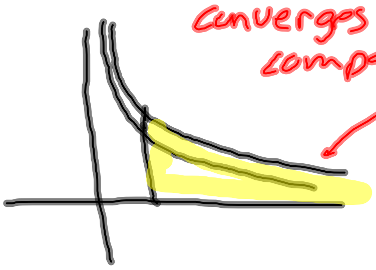
8.4 b Comparison test

$$0 < \int_1^\infty \frac{1}{x^3+1}\,dx < \int_1^\infty \frac{1}{x^3}\,dx = \frac{1}{2}$$

converges by comparison



$$\lim_{b\to\infty}\int_1^b x^{-3}\,dx$$

$$\lim_{b\to\infty} \left.\frac{x^{-2}}{-2}\right|_1^b$$

$$\lim_{b\to\infty} -\frac{1}{2b^2} - -\frac{1}{2} = \frac{1}{2}$$

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if $0 \le f(x) \le g(x)$

then $0 \le \int_a^\infty f(x)\,dx \le \int_a^\infty g(x)\,dx$

if diverges then $\int_a^\infty g(x)\,dx$ also diverges

If converges then $\int_a^\infty f(x)\,dx$ also converges

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$$\int_1^\infty \frac{1}{\sqrt{x^4+1}}\,dx < \int_1^\infty \frac{1}{\sqrt{x^4}}\,dx = \int_1^\infty \frac{1}{x^2}\,dx$$

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